# Scheme, More

# Pairs and Lists

# Pairs

> **cons: construct**
> **car and cdr: historical reason (Lisp on IBM 704)**

- Pairs are created using the **cons** expression in Scheme
- **car** selects the first element in a pair
- **cdr** selects the second element in a pair
- The second element of a pair must be another pair, or **nil (empty)**

```
scm> (define x (cons 1 (cons 3 nil)))
x
scm> x
(1 3)
scm> (car x)
1
scm> (cdr x)
(3)
```
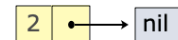
# Lists

(cons 2 nil)

- The only type of sequence in Scheme is the linked list
  - We can create these with pairs using multiple **cons** expressions
- **nil** represents the empty list

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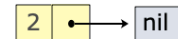


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>(define x (cons 1 (cons 2 nil))
>x

>(car x)

>(cdr x)

>(cons 1 (cons 2 (cons 3 (cons 4 nil))))

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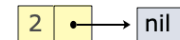
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(1 2 3 4)
```

Demo_1

# Symbolic Programming

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Quotation is used to refer to symbols directly in Lisp.

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Short for (quote a), (quote b): Special form to indicate that the expression itself is the value.

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Quotation can also be applied to combinations to form lists.

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>'(a b c)
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(b c)
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Demo_2

# Tail Recursion

# Recursion Versus Iteration in Python

```python
def rfactorial(n):
    if n == 0:
        return 1
    else:
        return n * rfactorial(n - 1)
def ifactorial(n):
    total = 1
    while n > 0:
        total *= n
        n -= 1
    return total
```

| Multiplication Operations? | Frames? |
|:---:|:---:|
| n | n |
| n | 1 |

Demo_3

# Tail Recursion

- We say an expression is in a **tail context** if it is evaluated as the last step in the function call
  - That means nothing is evaluated/applied after it is evaluated
- Function calls in a tail context are called **tail calls**
- If the tail call calls the function itself, we say that function is **tail recursive**
  - If a language supports tail call optimization, a tail recursive function will only ever open a constant number of frames

# Identifying Tail Contexts

An expression is in a tail context only if **it is the last thing evaluated** in every possible scenario (no other action is performed afterwards)

For each of the following expressions, which expressions (expr1, expr2, expr3) are in a tail context?

```
(and expr1 expr2 expr3)          (+ expr1 expr2)



(if expr1 expr2 expr3)           ((lambda (expr1) expr1) expr2)
```

# Recursive frames

```
(define (fact n)
  (if (= n 0)
    1
    (* n (fact (- n 1)))))
```

Consider a call to fact(4)

**f1:** fact
        **n:** 4
        **rv:** 24

**f2:** fact
        **n:** 3
        **rv:** 6

**f3:** fact
        **n:** 2
        **rv:** 2

**f4:** fact
        **n:** 1
        **rv:** 1

**f5:** fact
        **n:** 0
        **rv:** 1

We need to keep these frames open because the last step in the function is to multiply n with the result of the recursive call.

# Tail calls

```
(define (fact n)
  (define (fact-tail n result)
    (if (<= n 1)
        result
        (fact-tail (- n 1) (* n result)))))
  (fact-tail n 1))

fact(4)
```

Number of frames the same regardless of input size!

...ail
...r (fact-tail (- 4 1)
        (* 4 1)) returns

...ail
...r (fact-tail 2 12)

...ail
...r (fact-tail 1 24)

**f4:** fact-tail
**n:** 1
**result:** 24
**rv:** 24

# Writing Tail Recursive Functions

1) Identify recursive calls that are not in a tail context. Tail contexts are:

   ○ The last body subexpression in a *lambda* (a function)

   ○ The consequent and alternative in a tail context *if*

   ○ The last sub-expression in a tail context *and, or, begin,* or *let*

2) Create a helper function with arguments to accumulate the computation that

   prevents it from being tail recursive

# Example: Length of Linked List

Goal: Write a function that takes in a list and returns the length of the list. Make sure it is tail recursive.

```scheme
(define (length lst)
    (if (null? lst)
        0
        (+ 1 (length (cdr lst))))))

scm> (length '())
0
scm> (length '(1 2 (3 4))
3
```

```scheme
(define (length-tail lst)



)
```

Demo_5