Data Abstraction

Announcements

Data Abstraction

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- Data abstraction: A methodology by which functions enforce an abstraction barrier between ***representation*** and ***use***

Data Abstraction

All Programmers

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All Programmers

Great Programmers

Rational Numbers

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$$\frac{\text{numerator}}{\text{denominator}}$$

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Exact representation of fractions

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A pair of integers

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- `rational(n, d)` returns a rational number `x`

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Exact representation of fractions

A pair of integers

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Assume we can compose and decompose rational numbers:

- rational(n, d) returns a rational number x
- numer(x) returns the numerator of x

Rational Numbers

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Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- rational(n, d) returns a rational number x
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Assume we can compose and decompose rational numbers:

Constructor

- `rational(n, d)` returns a rational number x

Selectors

- `numer(x)` returns the numerator of x
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Rational Number Arithmetic

Example

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5}$$

Example

General Form

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$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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Example

$$\frac{nx}{dx} * \frac{ny}{dy}$$

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

Example

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

$$\frac{3}{2} + \frac{3}{5}$$

Example

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

General Form

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$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

General Form

Rational Number Arithmetic Implementation

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

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- `rational(n, d)` returns a rational number x
- `numer(x)` returns the numerator of x
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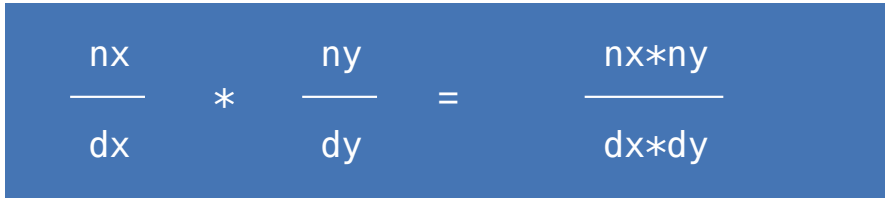
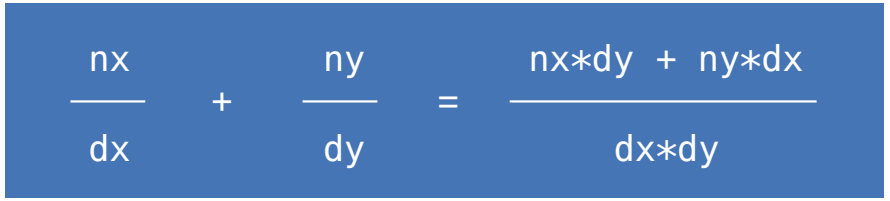
Rational Number Arithmetic Implementation

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))
```





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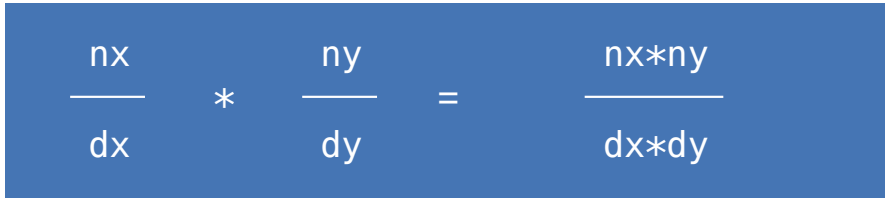

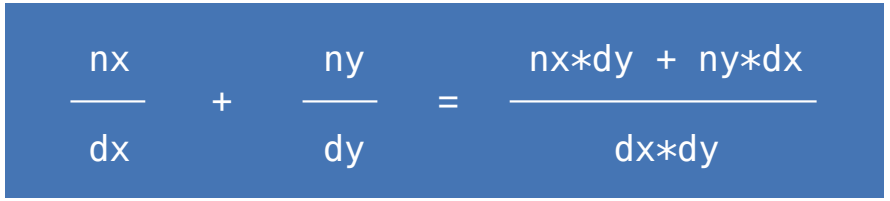

- rational(n, d) returns a rational number x
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These functions implement an abstract representation for rational numbers

Rational Number Arithmetic Implementation

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
                    denom(x) * denom(y))
```

Constructor

Selectors

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)
```

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

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Rational Number Arithmetic Implementation

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
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    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))
```

Constructor

Selectors

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

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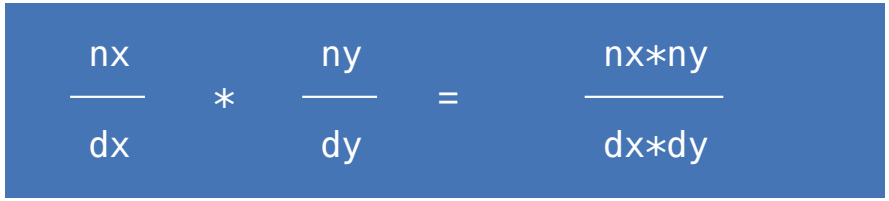
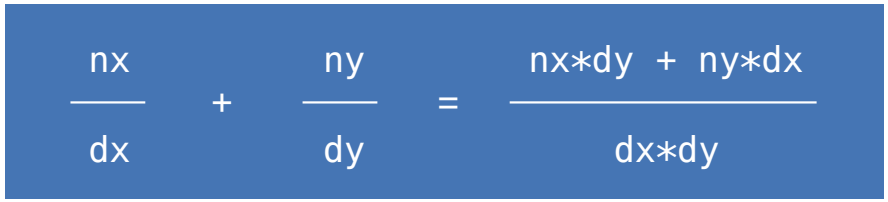



```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))

def rationals_are_equal(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```



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These functions implement an abstract representation for rational numbers

Pairs

Representing Pairs Using Lists

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```
>>> pair = [1, 2]
```

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```

A list literal:
Comma-separated expressions in brackets

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
```

A list literal:
Comma-separated expressions in brackets

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
```

A list literal:
Comma-separated expressions in brackets

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2
```

A list literal:
Comma-separated expressions in brackets

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Element selection using the selection operator

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2

>>> from operator import getitem
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Element selection using the selection operator

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]
```

A list literal:
Comma-separated expressions in brackets

```
>>> x, y = pair
>>> x
1
>>> y
2
```

"Unpacking" a list

```
>>> pair[0]
1
>>> pair[1]
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```

Element selection using the selection operator

```
>>> from operator import getitem
>>> getitem(pair, 0)
1
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>>> pair = [1, 2]
>>> pair
[1, 2]
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A list literal:
Comma-separated expressions in brackets

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[1, 2]
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A list literal:
Comma-separated expressions in brackets

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>>> x, y = pair
>>> x
1
>>> y
2
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"Unpacking" a list

```
>>> pair[0]
1
>>> pair[1]
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```

Element selection using the selection operator

```
>>> from operator import getitem
>>> getitem(pair, 0)
1
>>> getitem(pair, 1)
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```

Element selection function

Representing Rational Numbers

```
def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]
```

Representing Rational Numbers

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Construct a list

Representing Rational Numbers

```
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```

Construct a list

```
def numer(x):
    """Return the numerator of rational number X."""
    return x[0]
```

Representing Rational Numbers

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def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]
```

Construct a list

```
def numer(x):
    """Return the numerator of rational number X."""
    return x[0]

def denom(x):
    """Return the denominator of rational number X."""
    return x[1]
```

Representing Rational Numbers

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def rational(n, d):
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Construct a list

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def numer(x):
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    """Return the denominator of rational number X."""
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```

Select item from a list

Representing Rational Numbers

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def rational(n, d):
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def numer(x):
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Select item from a list

(Demo)

Reducing to Lowest Terms

Example:

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3}$$

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2}$$

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2}$$

$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2}$$

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2} \qquad \frac{2}{5} + \frac{1}{10}$$

$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2}$$

Reducing to Lowest Terms

Example:

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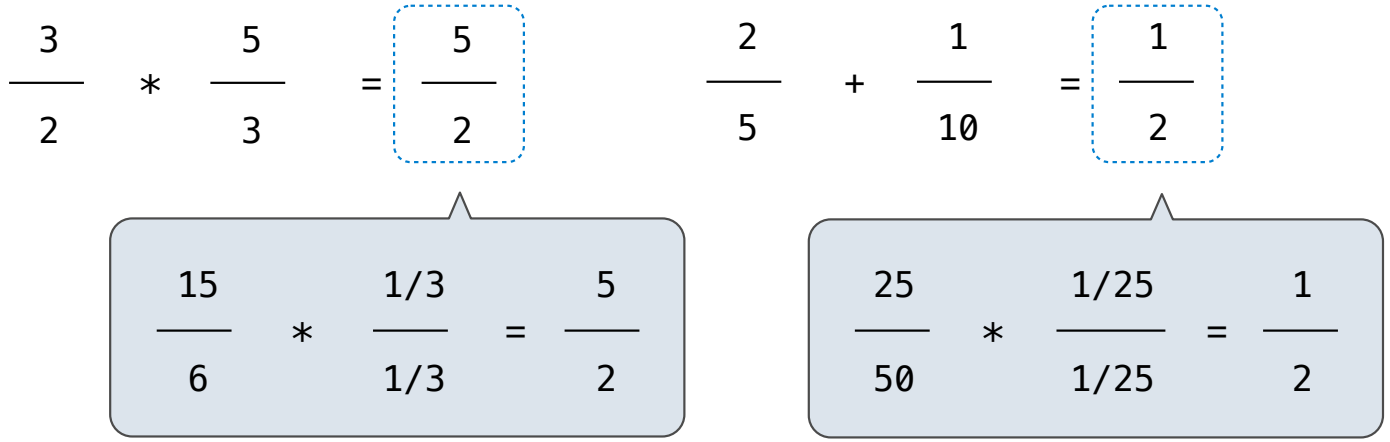
```
from fractions import gcd
```

Reducing to Lowest Terms

Example:




```
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def rational(n, d):
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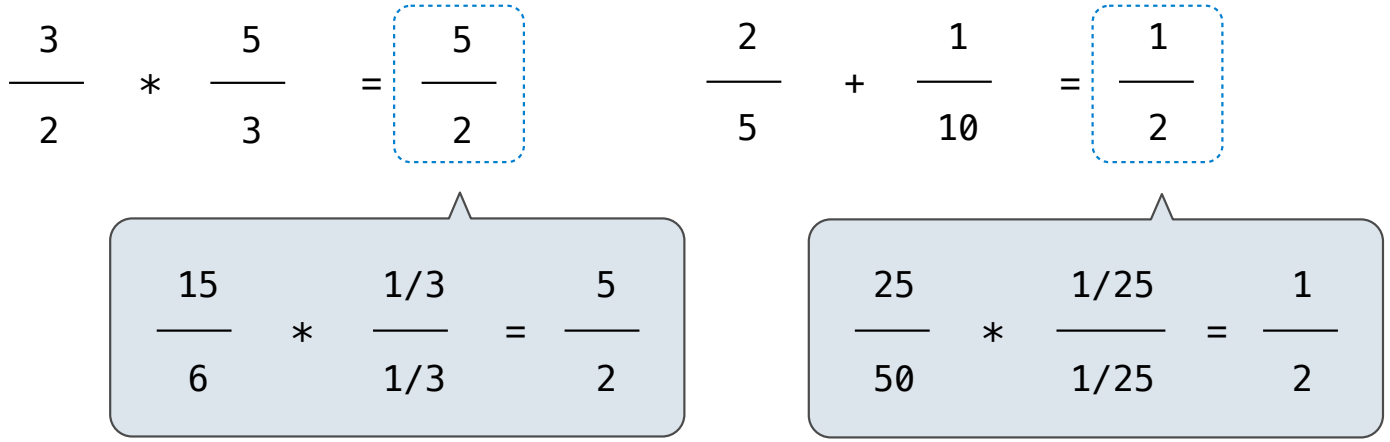
Reducing to Lowest Terms

Example:




```
from fractions import gcd

def rational(n, d):
    """Construct a rational that represents n/d in lowest terms."""
```

Reducing to Lowest Terms

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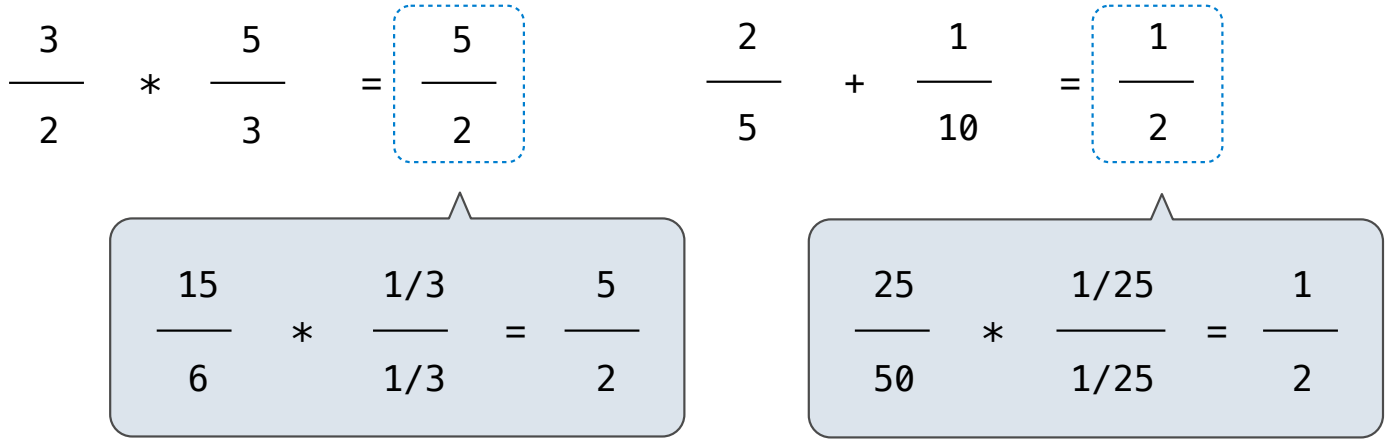


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Reducing to Lowest Terms

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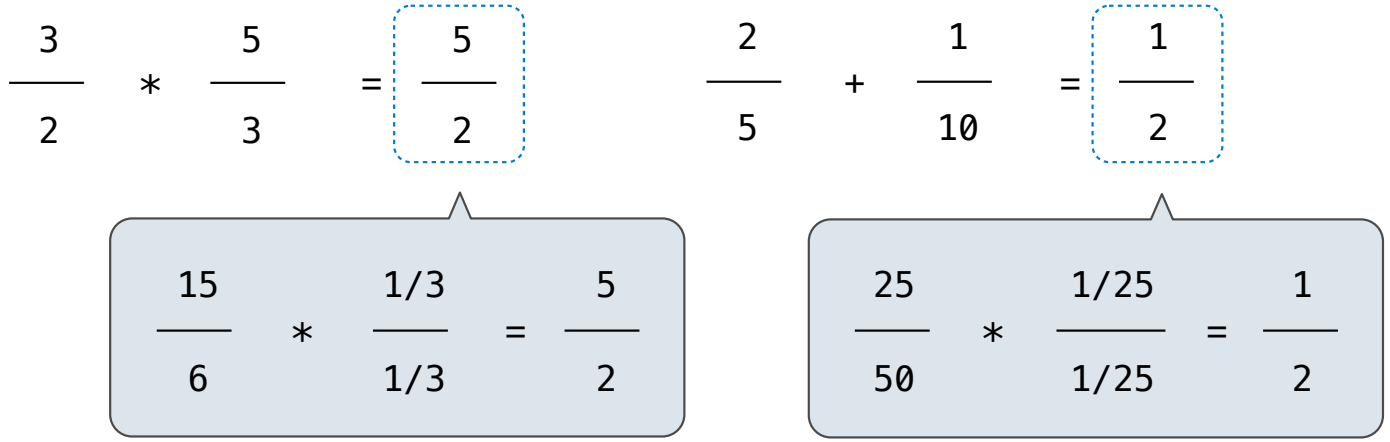


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from fractions import gcd

def rational(n, d):
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    g = gcd(n, d)
    return [n//g, d//g]
```

Reducing to Lowest Terms

Example:

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$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2} \qquad \frac{25}{50} * \frac{1/25}{1/25} = \frac{1}{2}$$

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Greatest common divisor

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(Demo)

Abstraction Barriers

Abstraction Barriers

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation		

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation	whole data values	

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Use rational numbers to perform computation	whole data values	`add_rational`, `mul_rational` `rationals_are_equal`, `print_rational`

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Parts of the program that...	Treat rationals as...	Using...
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Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
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Create rationals or implement rational operations	numerators and denominators	

Abstraction Barriers

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Implementation of lists

Violating Abstraction Barriers

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    return [ x[0] * y[1], x[1] * y[0] ]
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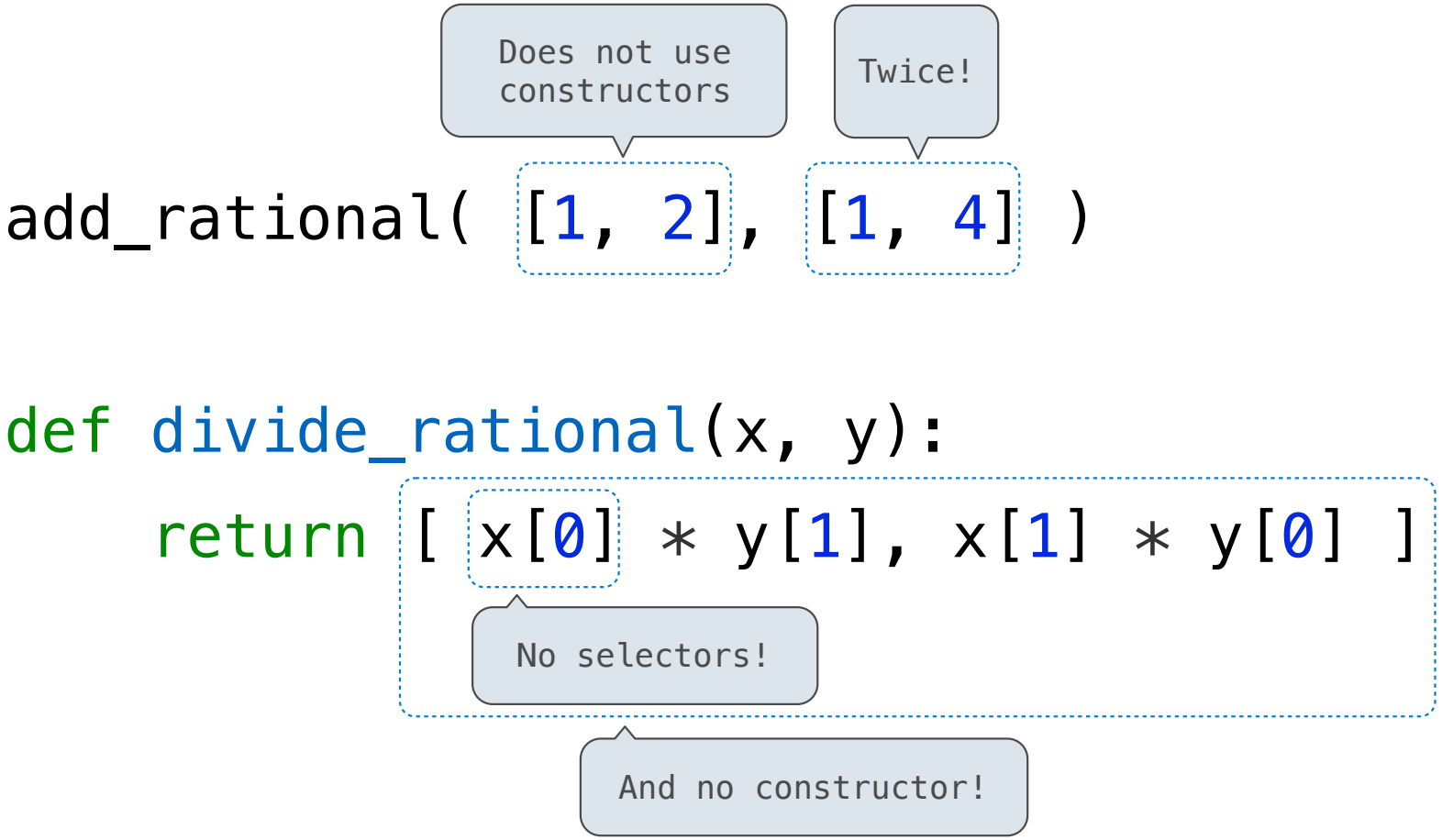

Violating Abstraction Barriers

Data Representations

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(Demo)

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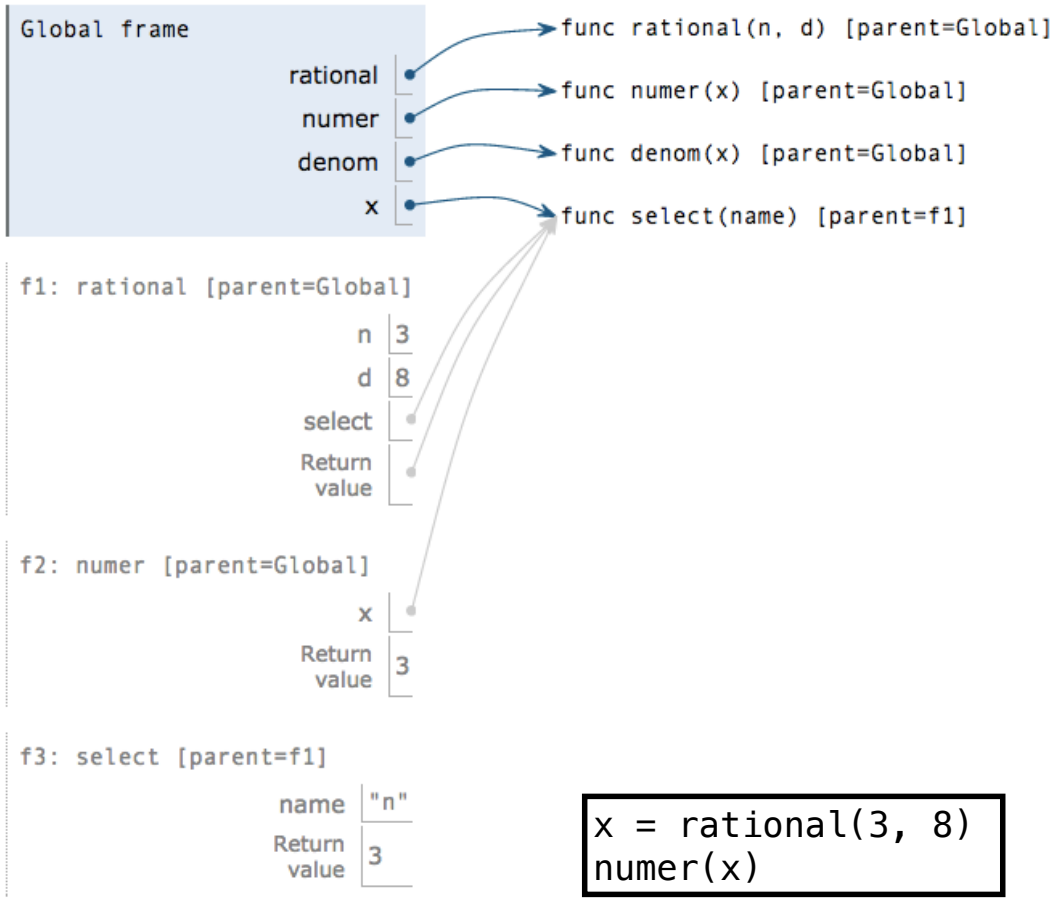


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